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Learning Iterative Image Reconstruction

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Abstract

Successful image reconstruction requires the recognition of a scene and the generation of a clean image of that scene. We propose to use recurrent neural networks for both analysis and synthesis.

The networks have a hierarchical architecture that represents images in multiple scales with different degrees of abstraction. The mapping between these representations is mediated by a local connection structure. We supply the networks with degraded images and train them to reconstruct the originals iteratively. This iterative reconstruction makes it possible to use partial results as context information to resolve ambiguities.

We demonstrate the power of the approach using three examples: superresolution, fill in of occluded parts, and noise removal / contrast enhancement.

1 Introduction

The quality of captured real world images is frequently not sufficient for the application at hand. The reasons for this can be found in the image formation process (e.g. occlusions) and in the capturing device (e.g. low resolution, sensor noise).

Goal of the reconstruction process is to improve the quality of measured images, e.g. by suppressing the noise. To separate noise from objects, models of the noise and the objects present in the images are needed. Then, the scene can be recognized and a clean image of that scene can be generated.

Hierarchical image decompositions using wavelets have been successfully applied to image denoising [Simoncelli and Adelson, 1996; Donoho and Johnstone, 1995]. The image is transformed into a multiscale representation and the statistics of the coefficients of this representation are used to threshold them. The back-projected images are then less noisy. Problematic with these approaches is that the choice of the wavelet transformation is usually fixed and the thresholding ignores dependencies between neighboring locations within a scale and between scales.

The recently proposed VISTA approach [Freeman and Pasztor, 1999] to learning low-level vision uses Markov random fields to model images and scenes. The parameters of these graphical models can be trained, e.g. for a superresolution task. However, the models have no hidden variables and the inference via belief propagation is only approximate.

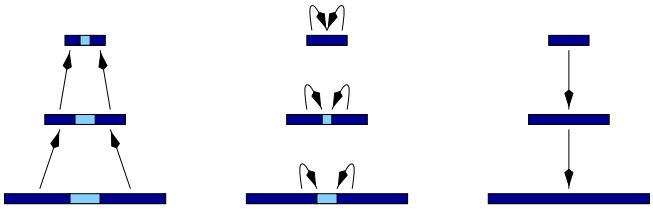

Figure 1: Iterative image reconstruction.

Continuous attractor networks have been proposed to complete images with occlusions [Seung, 1998]. For digits belonging to a common class, a two-layer recurrent network was trained using gradient descent to reconstruct the original. The network had many adaptable parameters, since no weight sharing was used. Further, it was not demonstrated that the reconstruction is possible, if the digit class is unknown. We extend the approach by adding lateral connections, weight sharing, and more layers to the network and train it to reconstruct digits from all classes without presenting the class label.

A common problem with image reconstruction is that it is difficult to decide locally about the interpretation of an image part. For example in a digit binarization task, it might be impossible to decide whether or not a pixel belongs to the foreground by looking only at the pixel's intensity. If contrast is low and noise is present, it could be necessary to bias this decision with the output of a line-detector for that location.

In general, to resolve such local ambiguities, a large context is needed, but feed-forward models that consider such a large context have many free parameters. They are therefore expensive to compute and difficult to train.

We propose to iteratively transform the image into a hierarchical representation and to use partial results as context. Figure 1 illustrates the propagation of information from regions that are interpreted easily to ambiguous regions. Further, we describe the reconstruction problem using examples of degraded images and desired output images and train a recurrent neural network of suitable structure to do the job.

The remainder of the paper is organized as follows: In the next section, the hierarchical architecture of the proposed recurrent networks is introduced. Section 3 discusses the supervised training of such networks. Experiments on three image reconstruction tasks are presented in Section 4.

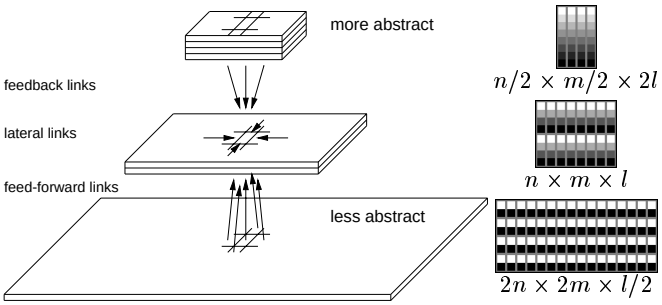

Figure 2: Sketch of the recurrent network.

2 Hierarchical Architecture

The neural abstraction pyramid architecture, introduced in [Behnke and Rojas, 1998], is a suitable framework for iterative image reconstruction.

The main features of the architecture are:

- *Pyramidal shape:* Layers of *columns* are arranged vertically to form a pyramid (see Fig. 2). Each column consists of a set of neural processing elements (*nodes*) with overlapping receptive fields. The number of nodes per column increases and the number of columns per layer decreases towards the top of the pyramid.
- *Analog representation:* Each layer describes an image in a two-dimensional representation where the level of abstraction increases with height, while spatial resolution decreases. The bottom layer stores the given image (a signal). Subsymbolic representations are present in intermediate layers, while the highest layers contain almost symbolic descriptions of the image content. These representations consist of *quantities* that have an *activity value* from a finite interval for each column.
- *Local interaction:* Each node is connected to some nodes from its neighborhood via directed *weighted links*. The shared weights of all nodes in a layer that represent the same quantity are described by a common *template*. The links can be classified as:
  - *feed-forward links:* perform feature extraction,
  - *lateral links:* for consistent interpretation,
  - *feedback links:* provide interpretation hypotheses.
- *Discrete time computation:* The update of a node's value for time step t depends only on the input values at $(t-1)$. All nodes are updated in parallel at each time step.

We use Σ-units as neural processing elements that compute the weighted sum of their inputs and apply a nonlinear output function. The update of the value $v_{x,y,z,q}$ of a unit at column (x, y) in layer z for quantity q is done as follows:

$$v^{t+1}_{x,y,z,q} = \sigma\left[\sum_{j\in\mathcal{L}(i)} \mathcal{W}(j)\, v^{t}_{\mathcal{X}(j,x),\mathcal{Y}(j,y),\mathcal{Z}(j,z),\mathcal{Q}(j)} + \mathcal{B}(i)\right].$$

The template $i = \mathcal{T}(z, q)$ is associated with quantity q at layer z. $\mathcal{L}(i)$ is the set of links of that template and $\mathcal{B}(i)$ is the template bias. $(\mathcal{X}(j, x), \mathcal{Y}(j, y), \mathcal{Z}(j, z), \mathcal{Q}(j))$ describe location and quantity of the input value for link j, and $\mathcal{W}(j)$ is the link weight. The output function $\sigma(x) = 1/(1 + e^{-x})$ is here a sigmoid function that limits the values to the interval $[0, 1]$. In addition to the weights and the bias a start value $\mathcal{V}^0(i)$ for initialization at $t = 0$ is needed for each template. The value of input nodes is set to a copy of the corresponding component of the input vector $\mathbf{x}_k$ of the current example k:

$$v^t_{x,y,z,q} = x^t_{k,\mathcal{I}(x,y,z,q)}, \text{ if } i = \mathcal{T}(z,q) \text{ is input template.}$$

The feed-forward inputs of a node come from all quantities in a small window at the corresponding position in the layer $(z - 1)$ directly below that node. Lateral connections link to all quantities in its neighborhood, including the node itself. Feedback links originate from the units in the layer above that correspond to the same position.

3 Training Recurrent Networks

In [Behnke, 1999] an unsupervised learning algorithm for the neural abstraction pyramid architecture has been proposed. It learns a hierarchy of increasingly abstract representations of the image content that could be used to improve the quality of the images. Here, we apply supervised training to achieve the desired image reconstruction.

Training of recurrent neural networks is difficult due to the non-linear dynamics of the system. Several supervised training methods have been proposed in the literature. Real-time recurrent learning (RTRL) [Williams and Zipser, 1989] is suitable for continuously running networks, but very resource intensive. The backpropagation through time algorithm (BPTT) [Williams and Peng, 1990] unfolds the network in time and applies the backpropagation idea to compute the gradient of the error function. Its computational costs are linear in the number of time steps the error is propagated back.

For image reconstruction, we present a static input $\mathbf{x}_k$ to the network and train it to quickly reach a fixed point that coincides with the desired output $\mathbf{y}_k$. Thus, the network runs only a few iterations and the gradient can be computed efficiently. No artificial truncation of history is necessary.

3.1 Objective Function

The goal of the training is to produce the desired output $\mathbf{y}_k$ as quickly as possible. To achieve this, the network is updated for a fixed number T of iterations. The output vector $\mathbf{v}^t_k$ collects the output units of the network in an appropriate order.

The output error δ^t_k, the difference between the activity of the output units $\mathbf{v}^t_k$ and the desired output $\mathbf{y}_k$ is not only computed at the end of the sequence, but after every update step. In the error function we weight the squared differences progressively, as the number of iterations increases:

$$E = \sum_{k=1}^{K}\sum_{t=1}^{T} t^2 \|\mathbf{y}_k - \mathbf{v}^t_k\|^2.$$

A quadratic weight t^2 has proven to give the later differences a large enough advantage over the earlier differences, such that the network prefers a longer approximation phase, if the final approximation to the desired output is closer.

The contribution of intermediate output values to the error function makes a slight modification to the original backpropagation rule necessary. At all copies of the output units $v^t_{x,y,z,q}$ for $t < T$ the difference $\delta^t_{k,I(x,y,z,q)}$ is computed and added to the backpropagated component of the gradient.

3.2 Robust Gradient Descent

Minimizing the error function with gradient descent faces the problem that the gradient in recurrent networks either vanishes or grows exponentially in time, depending on the magnitude of gains in loops [Bengio *et al.*, 1994]. It is therefore very difficult to determine a learning constant that allows for both stability and fast convergence.

For that reason, we decided to employ the RPROP algorithm [Riedmiller and Braun, 1993], that maintains a learning constant for each weight and uses only the sign of the gradient to determine the weight change. The learning rates are initialized to a moderate value, increased when consecutive steps have the same direction, and decreased otherwise. We modify not only the weights in this way, but adapt the bias and start values as well.

The RPROP training method proved experimentally to be much more stable than gradient descent with a fixed learning rate. However, to compute the gradient, all training examples have to be presented to the network, which is slow for large training sets. To accelerate the training we implemented the following modification. We use as batch only a small working set of training examples. This set is initialized at random. After each weight update, a small fraction of the examples is replaced with randomly chosen examples to ensure a stable estimate for the gradient that takes over time all training examples into account. With a working set of 1% of 60.000 training examples we realized a speedup of two orders of magnitude, as compared to the batch method, without compromising convergence.

4 Experimental Results

We conducted a series of experiments with images of handwritten digits to demonstrate the power of the proposed approach for iterative image reconstruction. The reason for choosing digits was that large datasets are publicly available and that the images contain multiscale structure which can be exploited by the learning algorithm. Clearly, if there were no structure to learn, the training would not help.

We degraded the digits by subsampling, occlusion, or noise and trained recurrent networks to reconstruct the originals.

4.1 Superresolution

For our first experiment we used the original NIST images of segmented binarized handwritten digits [Garris and Wilkinson, 1992]. The digits are given in a 128×128 window, but their bounding box is typically much smaller. For this reason, we centered the bounding box in a 64×64 window to produce the desired output Y. The input X to the network consists of 16×16 subsampled versions of the digits that have been produced by averaging 4×4 pixels.

The superresolution network has three layers, as shown in Figure 3. The low resolution image is input to the rightmost layer. Four 32×32 quantities represent the digit in the middle layer. They are connected to their 3×3-neighborhoods, to 2×2 windows of the output units, and to a single input node. The leftmost layer contains only the output units of the network. They are connected to four nodes in the middle layer and to their 3×3-neighborhoods.

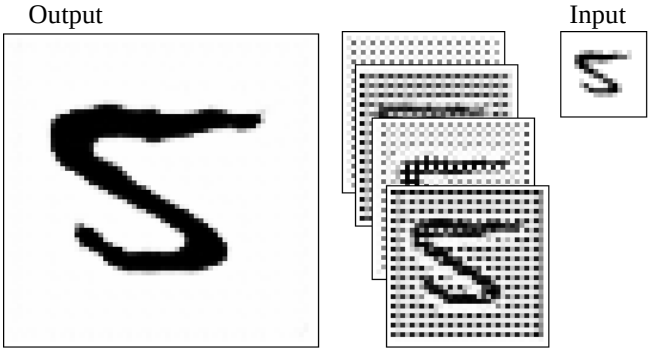

Figure 3: Network for superresolution.

We initialized the 235 free parameters of the network randomly and trained the network for ten time steps using 200 randomly chosen examples. As test set we used 200 different randomly chosen examples. Figure 4 shows for the first five test digits, how the output of the network develops over time. After two iterations the input can influence the output, but no further interactions are possible yet. In the following iterations the initial reconstruction is refined.

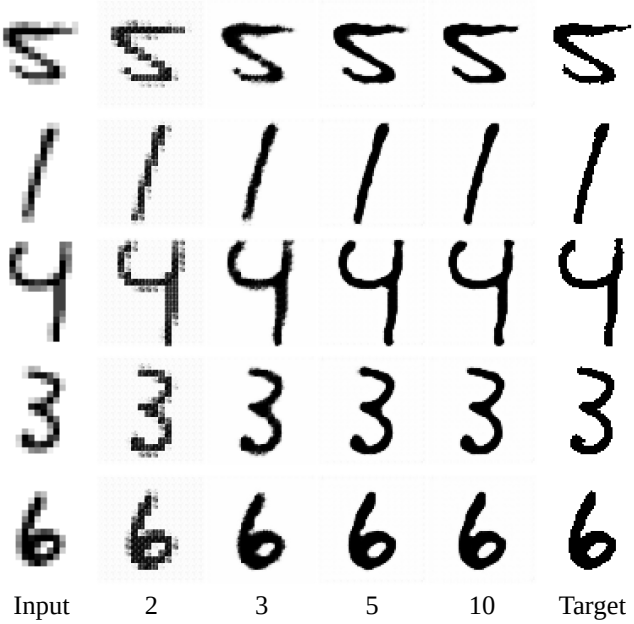

Figure 4: Iterative superresolution.

The network tries to concentrate the gray that is present in the input images at black lines with smooth borders. To illustrate this behavior, we presented uniform pixel noise to the network. The stable response after ten time steps is shown

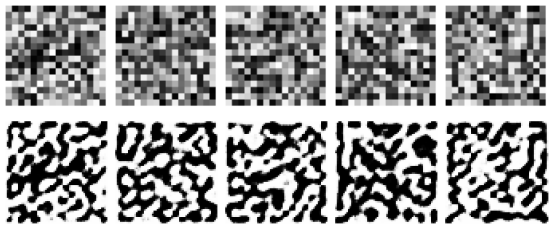

Figure 5: Response of the superresolution network to uniform noise.

in Figure 5. The network hallucinates smooth black lines at positions where many dark pixels are present.

We also trained a larger version of the recurrent network (RNN), that had eight hidden quantities in the middle layer as well as two feed forward neural networks (FFNN) with four and eight quantities. The units of the FFNNs looked at 3×3 windows of the previous layer such that the networks had a similar number of adjustable parameters as the corresponding RNNs. Figure 6 shows for the next five test digits the output of these four networks after 10 iterations. In general, the reconstructions are good approximations to the high resolution targets, given the low resolution inputs. The RNN outputs appear to be sharper than the responses of the FFNNs.

In Figure 7 the mean square error of the networks is displayed. The test set reconstruction error of the recurrent networks decreases quickly and remains below the error of the corresponding FFNN after six time steps. At iterations 9 and 10 the small RNN outperforms even the large FFNN.

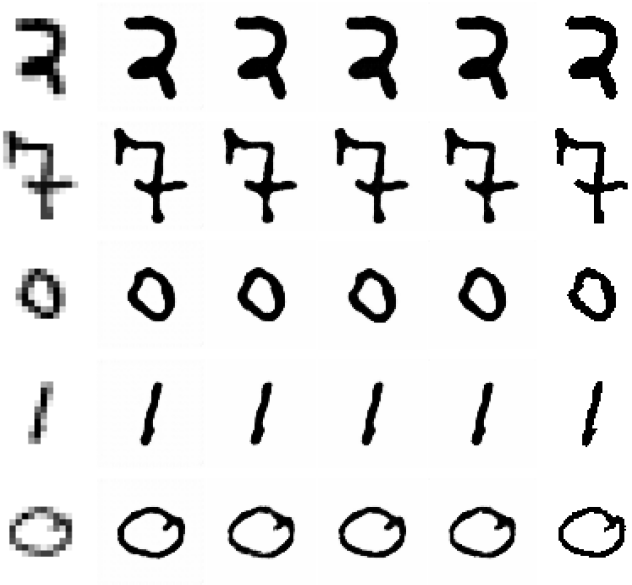

Figure 6: Outputs of different superresolution networks.

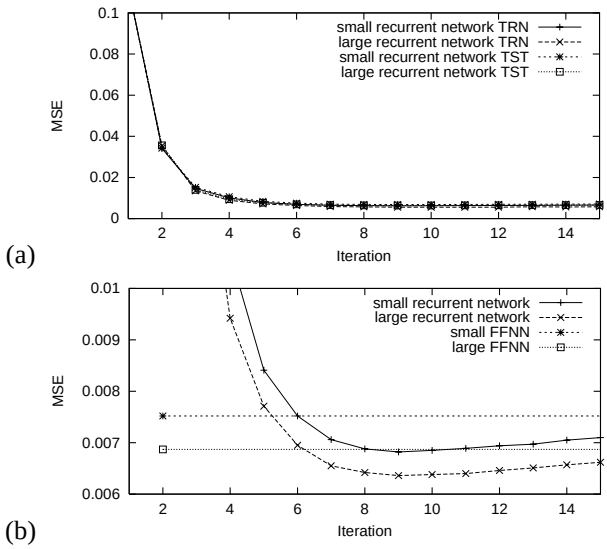

Figure 7: Mean square error of superresolution: (a) the recurrent network on the training set and the test set; (b) detailed view of the test set performance, compared to FFNN.

4.2 Fill In of Occluded Parts

For the second reconstruction experiment we used the MNIST database of handwritten digits [LeCun, 1994]. The NIST digits have been scaled to the size 20×20 and centered in an 28×28 image. We set an 8×8 square to the value 0.125 (light gray) to simulate an occlusion. The square was placed randomly at one of 12×12 positions, leaving a 4 pixel wide border that was never modified.

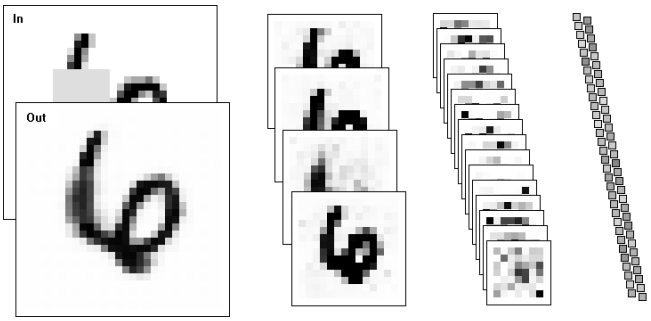

Figure 8: Network for fill in of occluded parts.

The reconstruction network consisted of four layers, as illustrated in Figure 8. The first layer (28×28) contains the input image and the output units of the network. In the second layer four quantities with resolution 14×14 look at overlapping 4×4 windows of the quantities below. The 16 quantities in the third layer have also 4×4 feed-forward connections, while in the top layer the resolution of the 64 quantities is reduced to 1×1 and the feed-forward weights are connected to all $7 \times 7 \times 16$ nodes of the third layer. The first three layers are surrounded by a one pixel wide border that is set to zero. In these layers the nodes have 3×3 lateral connections. In the fourth layer the lateral weights contact all 64 nodes. The feed-

back links are non-overlapping and have thus the size 2×2 between the first three layers and 7×7 between the third and the topmost layer.

Training is done with a working set of 600 of the 60.000 examples for twelve time steps. Figure 9 displays the reconstruction process for the first ten digits of the test set. One can observe that the images change mostly at occluded pixels. This shows that the network recognized the occluding square. Further, the change is such that a reasonable guess is produced, how the digit could look like behind the square. The network connects lines again that have been interrupted by the square. It is also able to extend shortened lines and to close opened loops. In most cases, the reconstructions are very similar to the original digits.

4.3 Noise Removal and Contrast Enhancement

The last experiment uses the same network architecture and the same MNIST digits, but degrades the input images as follows. We scaled the pixel intensities to $[0.25, 0.75]$, added a random background level that was uniformly distributed in the range $(-0.25, 0.25)$, and added uniform pixel noise in the range $(-0.25, 0.25)$. Finally, we clipped the pixel values at $[0, 1]$. The first column of Figure 10 shows the first ten digits of the test set that have been corrupted in this way. The network was trained on a working set of 600 out of 60.000 digits for twelve time steps.

The reconstruction process is also shown in Figure 10. One can observe that the network is able to detect the dark lines, to complete them, to remove the background clutter, and to enhance the contrast. The interpretation of most locations is decided quickly by the network. Ambiguous locations are kept for some iterations at intermediate values, such that the decision can be influenced by neighboring nodes. The reconstructed digits are very similar to the originals.

5 Discussion

The experiments demonstrated that difficult non-linear image reconstruction tasks can be learned by hierarchical neural networks with local connectivity. Supervised training of the networks was done by a combination of BPTT and RPROP.

The networks reconstruct images iteratively and are able to integrate partial results as context information for the resolution of local ambiguities. This is similar to the recently demonstrated belief propagation in graphical networks with cycles. The difference is that the proposed approach learns horizontal and vertical feedback loops that produce rich multiscale representations to model the images where current belief propagation approaches use either trees or arrays to represent the vertical or horizontal dependencies, respectively.

Further, the proposed network can be trained to compute an objective function directly, while inference in belief networks with cycles is only approximate due to multiple counting of the same evidence.

Recently, generalized belief propagation has been proposed [Yedidia *et al.*, 2001] that allows for better approximations of the inference process. It would be interesting to investigate the relationship between this approach and the proposed hierarchical recurrent neural networks.

The iterative reconstruction is not restricted to static images. The training method allows for a change of input and/or desired output at each time step. Thus, the networks should be able to integrate information over time, which would help to reconstruct video sequences.

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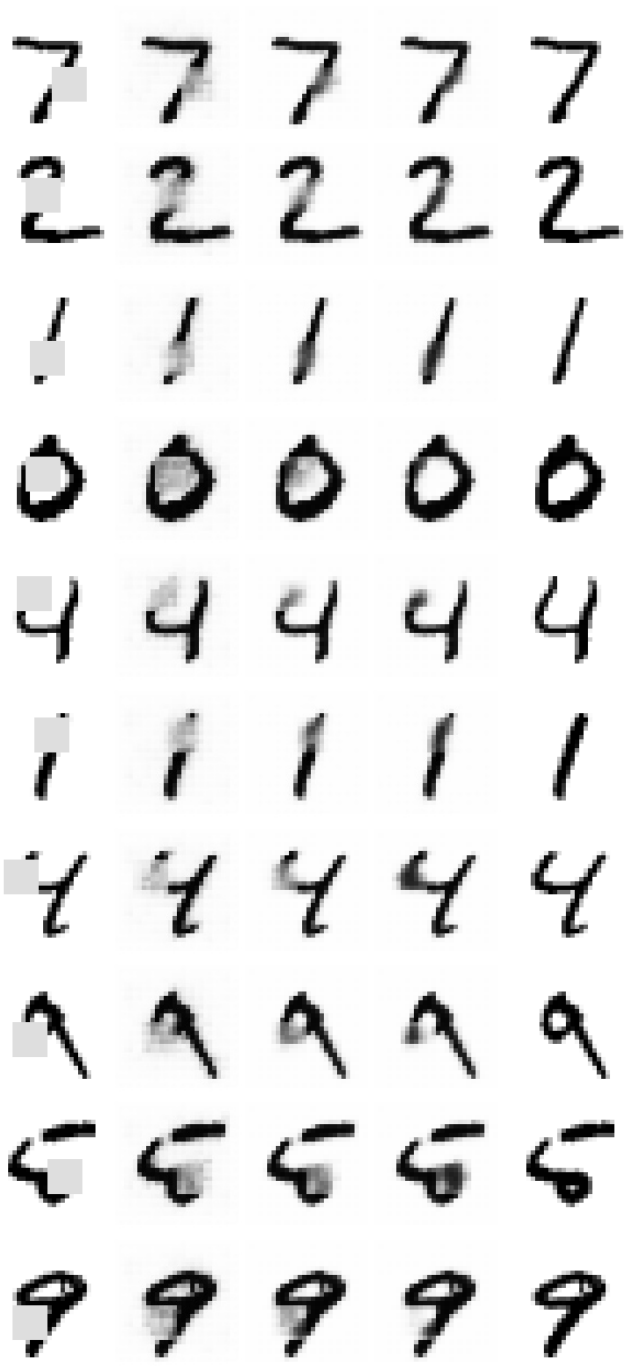

Figure 9: Fill in of occluded parts.

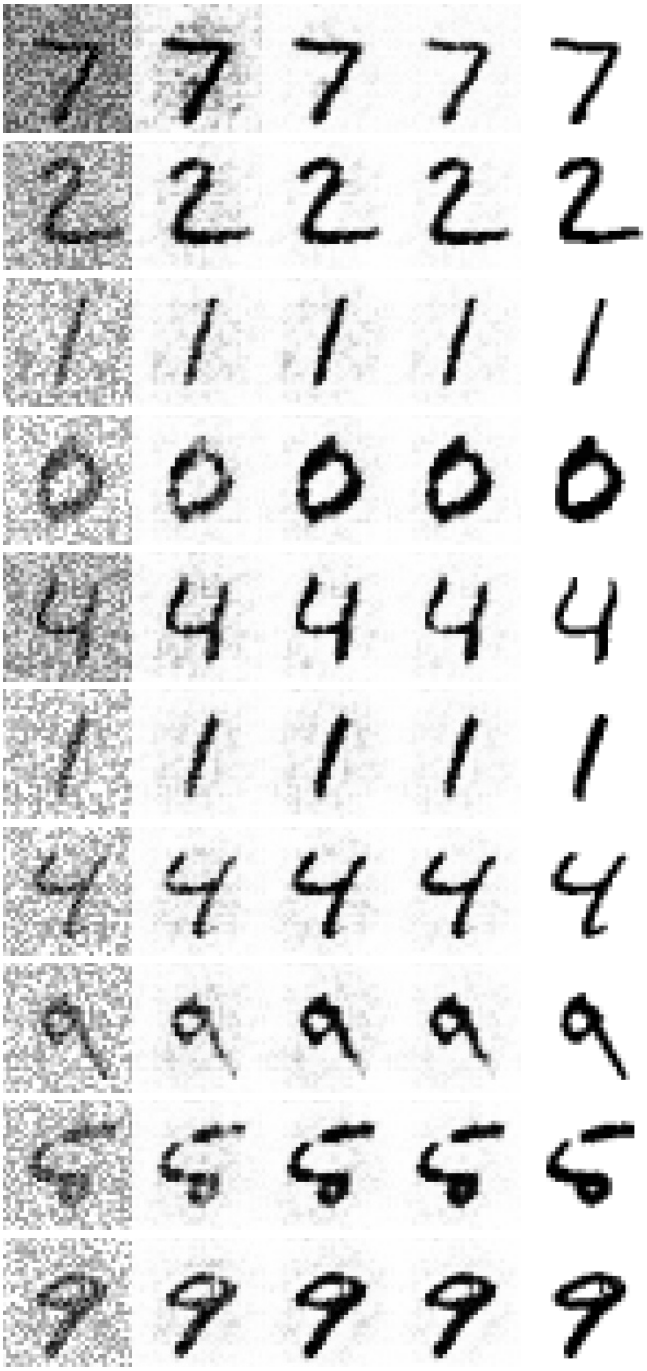

Figure 10: Noise removal and contrast enhancement.